

6. a. $(\{0,1\}, \{A, D\}, A, R)$ $R := \{A \rightarrow 01, A \rightarrow ABCD, B \rightarrow 111, D \rightarrow 00, C \rightarrow D, D \rightarrow A\}$

1. $Nu\{D_0, D_1\}$

- ersetze: $A \rightarrow 01 \Rightarrow A \rightarrow D_0 D_1$; $B \rightarrow 111 \Rightarrow B \rightarrow D_1 D_1 D_1$;
 $D \rightarrow 00 \Rightarrow D \rightarrow D_0 D_0$
 - neu: $D_0 \rightarrow 0, D_1 \rightarrow 1$

$\Rightarrow R_1 := \{A \rightarrow D_0 D_1, A \rightarrow ABCD, B \rightarrow D_1 D_1 D_1, D \rightarrow D_0 D_0, C \rightarrow D, D \rightarrow A, D_0 \rightarrow 0, D_1 \rightarrow 1\}$

2. $A_1 \Rightarrow A_2 \Rightarrow \dots \Rightarrow A_k \Rightarrow a$

~~$C \Rightarrow B \Rightarrow A \Rightarrow D_0 D_1$~~ nicht relevant
 $\Rightarrow D_0 D_1$

- $A_1 \Rightarrow A_2 \Rightarrow \dots \Rightarrow A_k \Rightarrow B_1 \dots B_n$ $k \geq 2$
 $C \Rightarrow D \Rightarrow A \Rightarrow D_0 D_1$
 $\Rightarrow ABCD$
 $\Rightarrow D_0 D_1$

$\Rightarrow$ fixe hinzu: $C \rightarrow D_0 D_1, C \rightarrow ABCD, C \rightarrow D_0 D_1, D \rightarrow D_0 D_1, D \rightarrow ABCD$
 - eliminiere: $C \rightarrow D, D \rightarrow A$

$\Rightarrow R_2 := \{A \rightarrow D_0 D_1, A \rightarrow ABCD, B \rightarrow D_1 D_1 D_1, D \rightarrow D_0 D_1, D_0 \rightarrow 0, D_1 \rightarrow 1, C \rightarrow D_0 D_1, C \rightarrow ABCD, C \rightarrow D_0 D_1, D \rightarrow D_0 D_1, D \rightarrow ABCD\}$

3. $A \rightarrow AE_1, E_1 \rightarrow BE_2, E_2 \rightarrow CD, B \rightarrow D_1 E_3, E_3 \rightarrow D_1 D_1, C \rightarrow AE_1, D \rightarrow AE_1$

$\Rightarrow G' = (\{0,1\}, \{A, B, C, D, D_0, D_1\}, A, R')$

$R' = \{A \rightarrow D_0 D_1, A \rightarrow AE_1, E_1 \rightarrow BE_2, E_2 \rightarrow CD, B \rightarrow D_1 E_3, E_3 \rightarrow D_1 D_1, D \rightarrow D_0 D_1, D_0 \rightarrow 0, D_1 \rightarrow 1, C \rightarrow D_0 D_1, C \rightarrow AE_1, C \rightarrow D_0 D_1, D \rightarrow D_0 D_1, D \rightarrow AE_1\}$

b. $(\{a,b,c\}, \{S, A, B, C, D\}, S, R)$ $R := \{S \rightarrow A, A \rightarrow C, C \rightarrow ABC, S \rightarrow a b c D, D \rightarrow a, B \rightarrow A, C \rightarrow S A B\}$

1. $Nu\{D_a, D_b, D_c\}$

- ersetze: $S \rightarrow D_a D_b D_c D, D \rightarrow D_a, C \rightarrow S A D_b$
 - neu: $D_a \rightarrow a, D_b \rightarrow b, D_c \rightarrow c$

2. $A_1 \Rightarrow A_2 \Rightarrow \dots \Rightarrow A_k \Rightarrow a$ $k \geq 2$: ~~hier~~
 ~~$S \rightarrow A \rightarrow C \rightarrow ABC$~~ $\Rightarrow$ Bsp. $D \rightarrow a$

- $A_1 \Rightarrow A_2 \Rightarrow \dots \Rightarrow A_k \Rightarrow B_1 \dots B_n$ $k \geq 2$
 $S \rightarrow A \rightarrow C \rightarrow ABC$
 $\rightarrow S A D_b$
 $B \rightarrow A \rightarrow C \rightarrow ABC$
 $\rightarrow S A D_b$
 Bsp: $S \rightarrow ABC, S \rightarrow S A D_b, A \rightarrow ABC, A \rightarrow S A D_b$
 $B \rightarrow ABC, B \rightarrow S A D_b$

- eliminiere $S \rightarrow A, A \rightarrow C, D \rightarrow D_a, B \rightarrow A$

3. $S \rightarrow D_a E_1, E_1 \rightarrow D_b E_2, E_2 \rightarrow D_c D, C \rightarrow A E_3, E_3 \rightarrow B C, C \rightarrow S E_4, E_4 \rightarrow A D_b, S \rightarrow A E_3, A \rightarrow A E_3, B \rightarrow A E_3, S \rightarrow S E_4, B \rightarrow S E_4, A \rightarrow S E_4$

$\Rightarrow G' = (\{a,b,c\}, \{S, A, B, C, D, D_a, D_b, D_c\}, S, R')$

$R' = \{D_a \rightarrow a, D_b \rightarrow b, D_c \rightarrow c, D \rightarrow a, S \rightarrow D_a E_1, E_1 \rightarrow D_b E_2, E_2 \rightarrow D_c D, C \rightarrow A E_3, E_3 \rightarrow B C, C \rightarrow S E_4, E_4 \rightarrow A D_b, S \rightarrow A E_3, A \rightarrow A E_3, B \rightarrow A E_3, S \rightarrow S E_4, B \rightarrow S E_4, A \rightarrow S E_4\}$

c. $(\{a, b, c\}, \{S, A\}, S, R) \models$
 $R = \{S \rightarrow aSb, S \rightarrow ab, S \rightarrow A, A \rightarrow c, A \rightarrow Ac\}$

1. - $NU \{D_a, D_b, D_c\}$
 - ersetze: $S \rightarrow D_a S D_b, S \rightarrow D_a D_b, A \rightarrow D_c, A \rightarrow A D_a$
 - ergänze: $D_a \rightarrow a, D_b \rightarrow b, D_c \rightarrow c$

2. - $A_1 \Rightarrow A_2 \Rightarrow \dots \Rightarrow A_k \Rightarrow a \quad k \geq 2$
 $S \rightarrow A \rightarrow D_c \rightarrow c \Rightarrow$ eg. $A \rightarrow c, S \rightarrow c$
 - $A_1 \Rightarrow A_2 \Rightarrow \dots \Rightarrow A_k \Rightarrow B_1 \dots B_m \quad k \geq 2, m \geq 2$
 $S \rightarrow A \rightarrow \text{AD}_c$ eg. $S \rightarrow A D_c$
 - eliminiere: $S \rightarrow A, A \rightarrow D_c$

3. - $S \rightarrow D_a E, E \rightarrow S D_b$

$G' = (\{a, b, c\}, \{S, A, D_a, D_b, D_c, E\}, S, R')$ $\models$
 $R' = \{S \rightarrow D_a D_b, A \rightarrow A D_a, D_a \rightarrow a, D_b \rightarrow b, D_c \rightarrow c, A \rightarrow c, S \rightarrow c, S \rightarrow A D_c, S \rightarrow D_a E, E \rightarrow S D_b\}$

8. Konstruktion kontextfreier Grammatik $\{a^n b^m \mid 1 \leq n \leq 2m\}$

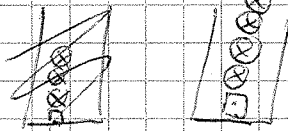
$aa bbb^* = aab b^+$ a gerade mindestens
 $aaa bbb^* = aaa b b^+$
 $aaaa bbb b^* = aaaa b b b^+$
 $aaaaa bbb b b^*$

$a^2 b^2 b^x$
 $a^3 b^2 b^4$
 $a^4 b^3 b^9$
 $a^5 b^3 b^9$

$a^n b^{\lfloor \frac{n}{2} \rfloor + 1} b^n$

$$L = \{a^n b^{\lfloor \frac{n}{2} \rfloor + 1} b^*\}$$

$\approx$



10. Kontextsensitiv: $uXv \rightarrow uvv, X \in N, uv \in (Z \cup N)^*, v \in (Z \cup N)^+$

$\alpha, G = (\{a\}, \{S, E, V\}, S, R) \quad R := \{S \rightarrow VS, S \rightarrow E, VE \rightarrow aE, VA \rightarrow aAV, E \rightarrow a\}$

$G' = (\{a\}, \{S, E, VA\}, S, R') \models$

$R' := \{S \rightarrow VS, S \rightarrow E, VE \rightarrow aE, VA \rightarrow AAV, E \rightarrow A, A \rightarrow a\}$

Problem: $VA \rightarrow AAV$:



$VA \rightarrow XAV \rightarrow XYV \rightarrow AYV \rightarrow AAV$

$VA \rightarrow D_1 A \rightarrow D_1 D_2 \rightarrow A A D_2 \rightarrow AAV$

$$L = \{a^{2^n} \mid n \in \mathbb{N}\}$$

$S \rightarrow VS$
 $\vdash$
 $ES E \rightarrow E V S E$